

One day in Michaelmas term, I was given a problem sheet which asked, among other things, for a homeomorphism between $\mathbb{Q} \times \mathbb{Q}$ and \mathbb{Q} . An artificial example is not hard to construct. But I was curious to know whether there was a more natural one – a polynomial in two variables with rational coefficients, for example. After some tinkering, I saw that I had no *a priori* reason to dismiss the possibility that a polynomial might do the job. But which polynomial? It was unclear. My thoughts turned to weaker conditions than being a homeomorphism. “Perhaps if I looked for an injective polynomial, I would find one easily.” But that was not the case.

A quick search revealed that Bjorn Poonen, now a renowned number theorist, had raised the same question that I raised about injectivity. There are few references on the subject apart from a paper of his, in which he proves that on condition of a certain conjecture on rational points on algebraic surfaces, there should be such an injective polynomial.

After discussing this briefly with Minhyong Kim, a tutor with whom I had shared many other ideas, I left it on the backburner.

But when the time came to think about undertaking a project over the summer, the matter resurfaced in my mind. Here was a fascinating yet largely unexplored problem, an opportunity to innovate while studying parts of mathematics that I enjoy, a taste of original research ... It felt right.

Much of the time was spent on independent reading. Though I had dabbled in algebraic and arithmetic geometry, the techniques that have been developed in the last two centuries are so advanced that there is always more to learn. I learnt a lot from Lorenzini’s “An Invitation to Arithmetic Geometry”, Silverman’s “Diophantine Geometry: An Introduction” and the numerous articles cited in these tomes. I also revisited material that I had encountered in my fresher year, but in which I had to have a solid foundation. This includes the courses given to students in Part B on algebraic number theory, Galois theory and algebraic curves.

I also tried my hand at Matlab, which was a new experience since it is not part of the curriculum for Mathematics and Philosophy. It turned out to be handy for the sort of numerical computation that was required. I was able to rule out large sets of polynomials simply by checking that they are not injective on $\mathbb{Z} \times \mathbb{Z}$. Towards the end of the summer, I had many fruitful discussions with computer scientists on the merits and demerits of different programs when it comes to Diophantine problems.

My project is a little unconventional in that I was not closely watched by my supervisor. Minhyong happened to be away in Korea between July and August, and we had agreed that independent reflection suited my style.

It is fair to say that throughout the duration of the project I was mired in personal problems, which made work and life difficult for me at times. Also, owing to circumstance, I shall have much scope for continuing my project with my tutor in the next two years. That is why I set aside the summer for the less ambitious aim of preparing the ground for any work on

arithmetic geometry in the future – an aim that is not to be undervalued, since there is so much ground to cover. The ultimate aim, however, is an article or publication setting out any partial results, if not an answer to the full conjecture.

I am honoured to have received a grant from the College for this project. It covered part of the cost of accommodation on Holywell Street. Neither the Mathematical Institute nor the LMS was able to provide me with a bursary. This, I imagine, is because it is hard for anything definite to be accomplished by the end of the vacation on the topic I have chosen. Nonetheless, the help given by the College was more than adequate.

It is my intention to go on to study for a PhD and do research on mathematics in my adult years. This project has been valuable to me in that regard. It has shown me how to take the first steps towards a difficult mathematical problem, and I hope that the work I have already done will lead to a publication and make for a useful addition to my CV.