# *The Merton Calculators*

Mathematics at Merton has a long and rich history, dating back almost as far as the foundation of the college itself in 1264. The earliest mathematicians of renown were a group of early $14^{th}$ century scholars known as the Oxford Calculators, also dubbed the Merton Calculators or the Merton School, because of their close connections with Merton College.

One of the most notable members of the Merton School was Thomas Bradwardine. Born in 1290 in Sussex, Bradwardine was educated at Balliol College, Oxford, and upon gaining his degree in 1321, he became a fellow of Merton College, where it is believed he remained until 1335. Whilst at Merton he gained an M.A. and a B.Th (Bachelor of Theology), and it was during this period that he wrote the majority of his works on mathematics, logic and philosophy. Probably the most famous of these was his 1328 work, *'Tractatus de proportionibus velocitatum in motibus'*, which translates roughly as 'treatise on the proportions of speeds in motions'. The fundamental problem Bradwardine was seeking to address in this work was determining how the speed of a moving body depends on the forces which act upon it – what effect does the relationship between the motive forces and the resistive forces have on the velocity?

At the time there were already several different propositions for the answer to this question, but Bradwardine believed that none of them provided a satisfactory explanation. A fundamental principle of Aristotle's natural philosophy states that motion can only occur when the motive forces acting on a body exceed the resistive forces – this intuitively makes sense, for example an object will only slide down a slope when the component of the object's weight in the direction of the slope is greater than the friction acting in the opposite direction, otherwise the object will stay put. The problem with some of the potential answers to Bradwardine's question was that they would violate this principle.

One unsatisfactory answer was given by Aristotle himself. Although it does not seem that he was intentionally trying to establish a law to relate speeds and forces, much of what is said in his works implies that the relation $V \propto F/R$ should hold, where V denotes the velocity of a body, F denotes the magnitude of the motive forces and R the magnitude of the resistive forces. At first this proposition may seem reasonable – greater motive forces lead to greater velocity, while a higher resistance should reduce the velocity. However, suppose that we fix the motive force F and take an initial resistive force $R_0$ such that $F>R_0$; then our object will be in motion with an initial velocity $V_0$. If the relationship $V \propto F/R$ holds and we consider continually halving the velocity, so that $V_1=0.5V_0$, $V_2=0.5V_1=0.25V_0$ and so on, this must correspond to continually doubling the resistive force so that $R_1=2R_0$, $R_2=2R_1=4R_0$ etc. Then there must be some t such that $R_t>F$, at which point we have velocity $V_t>0$ – but this contradicts the principle that motion will occur only when $F>R$, and so we conclude that $V \propto F/R$ is an unsuitable solution. Bradwardine similarly found contradictions to other proposed solutions such as $V \propto F - R$ and $V \propto (F-R)/R$.

So if these solutions aren't what we're looking for, what *is* the answer? The conclusion at which Bradwardine arrives is not the easiest to understand, because it is described in words and (quite apart from the fact that it's written in Latin!) some of the terminology he uses suggests something different to us now that what we believe he intended. However, if we translate it into modern notation we obtain something like the following. Rather than $V \propto F/R$, which suggests that in order to double the speed we must double the ratio of motive force to resistive force, Bradwardine seems to suggest that instead $V \propto \log(F/R)$, so that in order to double the speed we must square the ratio of motive force to resistive force. Most importantly, this formulation solves the problem we had with the other suggested solutions, that the principle that motion will occur only when $F>R$ is violated. As before, suppose we fix the motive force F and take an initial resistive force $R_0$ such that $F>R_0$; then our object will be in motion with an initial velocity $V_0$. If the relationship $V \propto \log(F/R)$ holds, then continually halving $V_0$ corresponds to continually taking square roots of $F/R_0$ – the crucial point being that if $F>R_0$, then

$F/R_0>1$, hence $(F/R_0)^{1/2}>1$, hence $F/R_1>1$ and so $F>R_1$, and we never have the problem of the resistive force exceeding the motive force and hence no contradiction. (I would here like to reiterate that this notation is nothing like what was used by Bradwardine, but essentially captures the same concepts and is used for clarity).

'Bradwardine's law' was widely accepted until the late 16th century, but we should note that it is in fact incorrect! Although the law does work in conjunction with Aristotle's laws of motion, these laws were themselves incorrect and hence Bradwardine's law does not accurately describe what happens in the real world. Nonetheless, Bradwardine's work was still a great step forward because it demonstrated the application of mathematics to problems of natural philosophy where previously this had not been considered. Up until this point, the subjects of mathematics and natural philosophy had been kept separate – Aristotle had disapproved of attempting to apply mathematical reasoning to problems of natural philosophy, believing that such deductions could not apply to other disciplines (which explains why there is very little mathematical calculation in his works on natural philosophy). Bradwardine however took the opposite view, believing that every physical law should be expressible in terms of mathematical functions, and this viewpoint opened the door to the systematic application of mathematics to previously intractable physical problems.

An interesting point to note about Bradwardine's work is that when we translate it into modern notation, we see that he utilises the concept of the logarithm (albeit implicitly) – considering that the logarithm as we know it was only invented by John Napier in the 17th century, this is another example of how Bradwardine was really ahead of his time in his thinking. (Important work about the logarithm was carried out by Mertonian Henry Briggs, see separate article!).

The other particularly famous result attributed to the Merton Calculators is the Mean Speed Theorem, which is also known as the Merton Mean Speed Theorem or the Merton Rule of uniform acceleration. This theorem is generally credited to William Heytesbury, who was a fellow of Merton from 1330, and is also known for his systematic application of logic to sophismata, or puzzling logical statements. The Mean Speed Theorem basically tells us that a body travelling with constant velocity v will travel the same distance in the same time as a body travelling with uniform acceleration from rest, provided that v is equal to half the final velocity of the accelerating body. This may sound a little confusing, so let's think about it in a different way.

Suppose that we have two students, Francesca and Thomas, who need to travel from Merton College to the Mathematical Institute for their morning lectures. They leave at the same time, and take the same route so that they travel the same distance. Thomas starts at rest and cycles with constant acceleration so that when he arrives at the lecture he is travelling with speed $v_T$. Francesca on the other hand likes to walk to lectures at a constant speed $v_F$, but she wants to meet Thomas there, so she consults the Mean Speed Theorem which tells her that if she chooses to walk at speed $v_F = \frac{1}{2}v_T$ then she will arrive at the same time as him.

Today we would express this rule in a different and more generalised way – if a body starts from initial velocity $v_0$ and accelerates uniformly in time $t$ to velocity $v_t$, then the distance $d$ it has travelled is given by an equation you might recognise: $d = \frac{1}{2}(v_0 + v_t)t$. Note that this result is very important, because of course it describes the motion of a body falling under gravity (neglecting air resistance of course!) and is thus the foundation of Galileo's famous Law of Falling Bodies. The Merton Calculators therefore made important advances in kinematics long before a more general and widely accepted theory was introduced in the 16th and 17th centuries – indeed, according to science historian Clifford Truesdell,

> *'The now published sources prove to us, beyond contention, that the main kinematical properties of uniformly accelerated motions, still attributed to Galileo by the physics texts, were discovered and proved by scholars of Merton college'* [1].

Undoubtedly the Merton Calculators paved the way for mathematicians investigating kinematics for centuries to come.