1 Introduction

This work was carried out remotely over the summer of 2020, with the laser accelerator research group, at the University of Oxford, funded by the Merton College Summer Project scheme.

Laser plasma wakefield accelerators are a novel way to create small and powerful tabletop accelerators. They consist of a high power short laser pulse being shot through a plasma, which creates a wakefield in the plasma, that can be used to accelerate particles.

2 Data analysis framework

Since experiments are expensive, and the amount of diagnostic data that can be gathered is limited, particle in cell (PIC) simulation software are used extensively in the study of laser accelerators. Multiple different simulation software are used, based on various principles, and using various output formats. This means that in order to analyse results, the code must be adapted to the given output file and data format, as well as geometry. To combat this problem a python library called PIC Analyser [1] was developed, with the aim to provide a powerful, common interface to analysing the output of PIC simulations, and provide many commonly used analysis functions.

The library requires input data in a very simple, unified format. To load the various data files, small adaptor modules can be made to convert the simulation data to this internal format. Currently a fully featured adaptor is provided to load cylindrical H5 files (those generated by FBPIC [2]), and a module is also provided to load data from SDF files (those generated by EPOCH).

This simple format is then converted to complex objects. In the case of fields, they are stored together with their geometry, and:

1. Support arithmetic operations (+,-,*,./,power)
2. Have a convenient interface for slicing and padding
3. Have (inverse) Fourier transform methods
4. Can be integrated
5. Name themselves based on operation history
6. Can be plotted
7. Have frequency space filtering functions

The source code of the library, along with detailed user instructions and examples can be found on the Oxford Physics GitLab: https://gitlab.physics.ox.ac.uk/mert4033/pic-analyser.

This framework was developed simultaneously with running some FBPIC [2] simulations. The simulation outputs were used to test the library, and the library was used to analyse the simulations.

3 wakefield isolation

In this simulation a single pulse was sent through a plasma, to excite a wakefield. The electric (and magnetic) fields are a sum of the laser’s electromagnetic wave, and the wakefield. The laser field is in a very specific frequency band, so they can easily be separated in Fourier space. Using the built in frequency space filtering, the separation was successful, and can be seen in Fig. 1.

![Figure 1: The original $E_z$ field (left) and the isolated wakefield (right) in a test simulation](image)

4 Photon acceleration

For this simulation, a high amplitude ($a_0 = 0.5$) driving pulse, followed by a small trailing pulse ($a_0 = 0.05$) were sent through a plasma. If the two pulses are out of phase (in our situation separated by 1.5 plasma wavelengths), the trailing pulse should be blue shifted and gain energy [3]. First we calculate the expected frequency shift:

The wakefield can simply be written as $n(z,t) = n_0 + \delta n \sin(k_p \zeta)$ with $\zeta = z - v_p t$, where in our case, the two beams are out of phase by 1.5 wavelength, so $k_p \zeta = 3\pi$. From Figure 2. we can see that $n_0 = 1.01 \cdot 10^{24} \text{m}^{-3}$, $\delta n = 0.11 \cdot 10^{24} \text{m}^{-3}$.

The wavelength of the laser is $\lambda = 800\text{nm}$, so $k_0 = 7853981 \text{m}^{-1}$, $\omega_0 = 2.355 \cdot 10^{15} \text{s}^{-1}$.

The expected frequency shift, taken from Equation 4 of [3]:

$$\Delta \omega = \frac{\omega_0^2}{2 \omega_0 n_0} k_p \Delta z \cos(k_p \zeta)$$
Figure 2: The electric field of the laser pulses (blue, left axis) vs the electron density (orange, right axis) plotted at the centre ($r = 0$) of the simulation

Here, $\omega_p = \sqrt{\frac{n_0 e^2}{\varepsilon_0 m_e}} = 5.6695 \cdot 10^{13} \text{s}^{-1}$ and $k_p = 189117 \text{m}^{-1}$ are the plasma frequency and wavelength. As a sanity check, this means the plasma wavelength is 33.2$\mu$m, which is realistic given the plot. So, the relative frequency shift for a given length:

$$\frac{\Delta \omega}{\omega_0 \Delta z} = 5.97 \text{m}^{-1}$$

Figure 3: High resolution (red, 80 samples per wavelength), medium resolution (green, 40) and low resolution (blue, 20) simulation results, for trailing pulse energy (left) and relative frequency (right)

So in the simulation length of 1cm, a frequency shift of 5.97% is expected, but in simulations we observe energy and frequency shifts on the order of 2%, as seen on Figure 3. On the energy plot we can clearly see that the low resolution simulation shows less energy transferred to the trailing pulse, likely due to numerical problems with either the simulation or the evaluation. The medium resolution simulation is much closer to the high resolution, indicating that a resolution in the range of 60-100 samples per wavelength is ideal for this particular experiment.

As a more general point, we can say that when choosing the resolution if a simulation, great care must be taken to choose a resolution where the effect is observable and numerically stable.
5 Energy recovery

In this final (simulated) experiment, we attempt to damp the wakefield with a second laser pulse. This is important, as in real accelerators, it is important to achieve a high repeated fire rate. Thus it is important to clean the plasma so the next accelerated bunch sees ideal conditions. It is also important to extract the remaining energy from the plasma, so the accelerating cavity doesn’t overheat during repeated shots.

In an ideal linear theory, if two pulses are sent into a plasma, the resulting wakefield is the superposition of their two individual wakes. So, if we send in two identical pulses, out of phase, we expect the wake to completely cancel. Since the second pulse is actually travelling in the wake of the first, non-linearities emerge, and the cancellation is not perfect. In our simulation, two pulses of $a_0 = 0.5$ were used. To make energy calculations easier, the pulses were polarised at different angles. The setup can be seen on Fig. 4.

![Figure 4: Driving pulse (x polarised, $E_x$, blue, left axis), trailing pulse (y polarised, $E_y$, orange, left axis), electron density (green, right axis) for the wakefield cleaning simulation. The red vertical line indicates the halfway point between the two pulses. The blue and orange vertical lines indicate the regions associated with the pulses for energy calculation.](image)

As we can see on Fig. 5, the wakefield is dampened, but far from completely cleaned. We expect the driving pulse to be loosing energy, as it is creating a wakefield, however the trailing pulse is reducing the amplitude of the wake, and hence should be gaining energy. On Fig. 6 we can see that this is not the case. The kinetic energy of the plasma in the simulation windows is only on the order of 0.5mJ, so it doesn’t contain a significant amount of energy. It is well known that PIC codes aren’t great at conserving energy, due to numerical problems. So it is possible that the trailing pulse gains energy at roughly the same rate as it is lost to numerical error, hence it maintains a nearly constant energy.

On the energy graph we can also see an oscillatory behaviour. This is also present in the wakefield, as the damping exhibits a periodic behaviour in time.

Further investigation could try experimenting with using a larger or smaller trailing pulse, or different phase shifts in order to optimise the wakefield cleaning. A pulse train could also be used to clean up the wake bit by bit with many small pulses.
Figure 5: The $E_z$ field for the wakefield cleaning experiment. The reduction in the wake thanks to the second pulse is noticeable, but far from perfect.

Figure 6: The energies of the trailing and driving pulses

6 Summary

A lightweight and powerful python library was developed to simplify the analysis of PIC simulation outputs. It was used to investigate photon acceleration, and found that effect is of a similar magnitude, but less than would be expected from theory. Then, two identical pulses were used to investigate the possibility of cleaning the wakefield. The effect is noticeable, but nowhere near perfect, probably due to the non-linear nature of the plasma.

7 References

References
