Henry Briggs

What is the decimal expansion of $\sqrt{5}$? How large is 99^7? $\sqrt[13]{2361}$? These might sound like rather bland questions to you, pressing a few buttons immediately gives the answer. But how would you resolve this problem if you had no calculator or computer? After all, this technology has only been around for a few decades. You might have heard of logarithm tables. Over centuries they were used to simplify calculations. Multiplication becomes addition, taking roots becomes division. The properties of logarithms might be very familiar to you, but have you ever wondered who created these tables?

One of these table makers was Henry Briggs. Not much is known his life in the 16th and 17th century. He is said to have been a very modest and devout man, fitting his life's work of creating tables through tireless effort, first at Gresham College in London, later at Merton.

Briggs was the very first Professor for Geometry at Gresham College, a position he kept for more than two decades. At the newly founded Gresham mariners met to learn nautical techniques. Public lectures in Geometry, Astronomy, Theology, Rhetoric, Music, Law and Medicine were held and quickly Gresham became the scientific center of its time. Briggs himself popularized the use of decimals, which were not commonly used before. His first tables, related to navigation, were also created during this time. However, in the early 17th century logarithms were discovered and Henry Briggs first encountered the work of John Napier. Napier's logarithms did not yet have the properties we associate with logarithms today. Instead, the identity $\log(ab) = \log(a) + \log(b) - \log(1)$ held true. $\log(1)$ was not equal to 0, instead it was a very large number. Briggs himself suggested to take $\log(1) = 0$ and use 10 as a basis, conventions that are far more familiar to us. Briggs' logarithm tables were created later, when he was already a fellow of Merton College.

With Gresham College as a scientific center, you might wonder which role Oxford and Cambridge played during these times. Well, in some sense the old universities were in a crisis. Scientific progress was rarely made; geometry and astronomy were barely taught and Euclid's elements were practically unknown. However, within a short time this would change. Two fellows of Merton, Thomas Bodley and Henry Savile played a significant role in this development. Bodley went on to found the Bodleian Library, while Savile, who became Warden, began to expand Merton's library. The number of books increased rapidly from 300 to 1595. Savile, also a mathematician, started teaching mechanics, optics and trigonometry. He then endowed the Savilian Chairs of Geometry and Astronomy. The first Savilian Professor of Geometry became Henry Briggs, who sensed the change that was about to happen and left Gresham for Merton. The remaining days of his life he devoted to creating new logarithm tables.

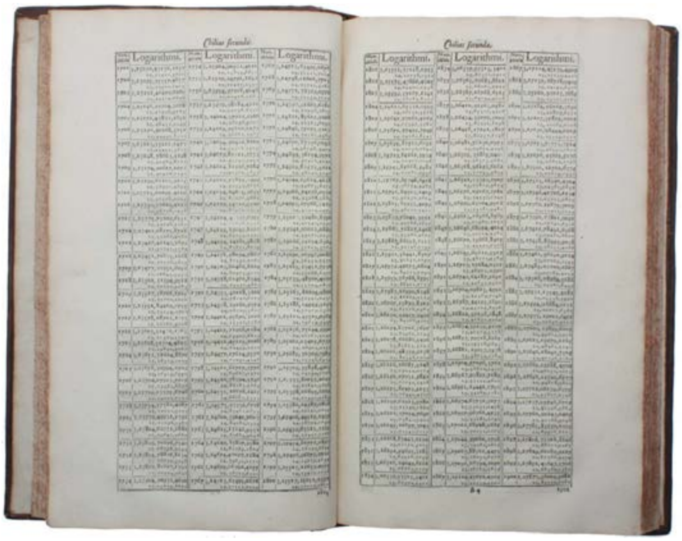
Having a look around Merton, you can spot objects dating back to Briggs' time at Merton. The sundial on the Chapel's wall and an instrument to measure the declination of the sun in Fellows Quad are thought to have been devised by either Briggs himself or the Savilian Professor of Astronomy of his time, John Bainbridge. However, the sundial has a flaw - it lies in the shadows for most of the day. Finally, Briggs' rather modest gravestone can be found in the Chapel.

Logarithms

In his Arithmetica Logarithmica Briggs calculated the logarithms of the numbers 1 to 20000 and 90001 and 100000 in base 10 up to fourteen decimal digits.

He started by calculating the logarithms of $\sqrt[2^n]{10}$ up to $n = 54$. Since $\log(\sqrt[2^n]{10}) = 2^{-n}$, these are easy to compute. For $x = 1 + r$ close to 1, Briggs found that $\log(x) \approx \alpha r$ where α is a constant. $\alpha \approx 0.4343$. Briggs evaluated this constant accurate to 16 decimal places. The exact value he sought was $\alpha = 1/(\ln(10))$. Now consider larger values of x. For large enough n we will have $\sqrt[2^n]{x} = 1 + r$ close enough to 1 so that we can use our approximation. Then $\log(1 + r) \approx \alpha r$ implies $\log x \approx 2^n \alpha r$. Hence successive square root extraction can be used to evaluate logarithms.

Obviously, this is a rather laborious process. Try it yourself for one or two numbers! Below you can see two pages of the Arithmetica Logarithmica. Briggs calculated some logarithms for up to 30 decimal places.



Magnetic Dip

While Briggs is mostly known for his work on logarithms, these were by far not the only tables he made. If you are interested in Geometry, you might like this approach to determining latitude.

In the 16th and 17th century navigation on the sea was difficult and sailors heavily relied on celestial bodies, for example by measuring the angle between sun and horizon or the position of the north star, in determining latitude. Of course, these methods heavily depended on the weather conditions. Robert Norman first discovered that while non-magnetized needles could be aligned with the horizon, magnetized needles would always make an angle with the horizontal. William Gilbert hoped to find a relation between latitude and magnetic dip, hence creating a new method for navigation on sea not relying on the sky.

Today we know that the magnetic field varies over time and that Gilbert's theory is incorrect, but this was not known at that time and led to the creation of Briggs' first tables. Below you can see the geometric construction he used for this. It is based on Gilbert's hypotheses that there is no dip at the equator point, at the pole the dip equals 90° and at 45° latitude the needle points to the second equator point.

In the sketch below the angle $\angle BAR$ gives the latitude. The angle $\angle TRA$ is the complement of the angle of dip. You can try to find a formula for $\angle TRA$ yourself! Here are a few hints and some necessary information: For simplification, since radius does not affect angles, you can assume $\overline{AB} = 1$. Per construction $\overline{BN}$ and $\overline{BC}$ have equal length. Furthermore, the arc DT has $\alpha/90$-times the length of arc DN. Here α denotes the value of the angle $\angle BAR$ in degree. While not necessary for obtaining a correct sketch, M, X, Y and Z are important for the calculation. You can apply the chord theorem. Other things you might need are Pythagoras, Sine Rule and some simple trigonometry.

Solution: We use the notation from the sketch below. For simplicity we also use $\overline{XY}$ to denote the length of the line connecting X and Y. Denote the angle $\angle BAR$ by α. Considering the triangle RAN, by the Sine Rule we find

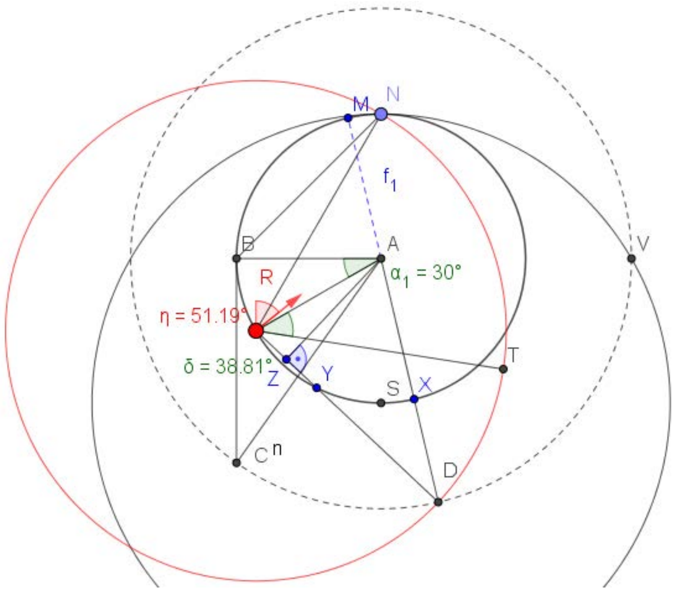
$$\frac{\overline{RN}}{\sin(90^\circ + \alpha)} = \frac{\overline{AN}}{\sin(45^\circ - \alpha/2)} = \frac{1}{\sin(45^\circ - \alpha/2)}.$$

(1) By Pythagoras' Theorem $\overline{BC}^2 = \overline{BN}^2 = 2\overline{AB}^2$. Further, $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 = 3\overline{AB}^2$. Since we took the radius of the circle to be 1, we now have $\overline{AC} = \overline{AD} = \sqrt{3}$. Applying chord theorem to M, X, Y, Z, D we obtain $\overline{MD} \cdot \overline{DX} = \overline{RD} \cdot \overline{DY}$. Then $(\overline{AD} + \overline{AB}) \cdot (\overline{AD} - \overline{AB}) = \overline{RN} \cdot \overline{DY}$ and $(\sqrt{3}+1)(\sqrt{3}-1) = \overline{RN} \cdot \overline{DY}$. We already found $\overline{RN}$ using the sine rule, so now we can evaluate $\overline{DY}$. We can also see that $\overline{RZ} = (\overline{RN} - \overline{DY})/2$ and $\angle RAZ = \sin^{-1}(\overline{RZ}/\overline{AR}) = \sin^{-1}(\overline{RZ})$.

(2) Then $\angle ARZ = 90^\circ - \angle RAZ$ and $\angle NRD = 45^\circ + \angle ARZ - \alpha/2$. By the relation between the arc length of DT and DN stated above we have $\angle TRD = \alpha/90 \cdot \angle NRD$. Also, $\angle TRA = \angle ARZ - \angle TRD$. We are looking for the complement of this angle, $\delta = 90^\circ - \angle TRA$. Combining all of these results, we have

$$\delta = \sin^{-1}(\overline{RZ}) + \frac{\alpha}{90} \cdot \left(135^\circ - \frac{\alpha}{2} - \sin^{-1}(\overline{RZ})\right),$$

where $\overline{RZ}$ can easily be found using paragraph (1).



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References

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[2] Roegel Denis, *A reconstruction of the tables of Briggs Artihmetica logarithmica.* 2010.